4. G. N. Dul'nev and Yu. P. Zarichnyak, Thermal Conductivity of Mixtures and Composite Materials [in Russian], Énergiya, Leningrad (1974).
5. A. S. Skal and B. I. Shklovskii, "Topology of an infinite cluster in flow theory and theory of jump conductivity," Fiz. Tekh. Poluprovodn., 8, 1586-1591 (1974).
6. G. N. Dul'nev, V. K. Kruglikov, and E. V. Sakhova, "Mathematical modeling of heterogeneous isotropic systems," Inzh.-Fiz. Zh., 41, No. 5, 859-864 (1981).

KINETICS OF THE SELF-SIMILAR CONDITIONS OF DIFFUSIONAL COMBUSTION OF POLYDISPERSE LIQUID FUEL

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UDC 621.1.016:536.46

On the basis of the kinetic equation for the particle distribution function with respect to the radius, a method is proposed for calculating the processes of heat and mass transfer in the combustion of liquid fuel of a polydisperse melt.

In the ignition of a liquid-drop or dustlike fuel in various devices, the determining role is played by the kinetics of particle combustion in a medium of oxidant diluted with inert gas. The problems arising here are very complex in view of their nonlinearity, and therefore they are solved, especially for the case of various engineering devices, by means of experimental methods or numerical calculations using a computer. In the latter case, the calculation of the combustion kinetics of a polydisperse particle system is undertaken by dividing the initial particle size distribution into narrow fractions [1-3]. An alternative approach which has been successfully used in calculations of vaporization and solution [4-6] is to use the kinetic equation for the particle distribution with respect to the radius.

In the present work, on the basis of this approach [5, 6], the simplest model of quasidiffusional combustion of polydisperse liquid-fuel drops in a volume with adiabatic walls is considered.

As in [6], it is assumed that the fuel drops are spherical in form; that their mass concentration is small; that the product consumption corresponds to stoichiometry of the reaction; and that there is no breakdown or coagulation of the drops; that the fuel is injected into gas containing oxidant with a temperature of the medium $T_{\mathrm{me.0}}$ above the ignition temperature. The assumptions adopted also hold in real conditions.

Polydispersity of the drops in the combustion process will be taken into account by the kinetic equation for the particle distribution function with respect to the radius $f(r_S, t)$ [5, 6]

$$\frac{\partial f(r_S,\ t)}{\partial t}+\frac{\partial}{\partial r_S}\left[f(r_S,\ t)\,W(r_S,\ t)\right]=0, \tag{1}$$

for which the following relation will hold

$$dN=N_0 f(r_S,\ t)\,dr_S;\quad f(r_S,\ 0)=f_0(r_S);\quad \int\limits_0^\infty f_0(r_S)\,dr_S=1. \tag{2}$$

In [6], the distribution function $f(r_S, t)$ for self-similar quasisteady conditions of evaporation was obtained in general form. Analogous determination of $f(r_S, t)$ is possible in combustion if the combustion rate of a single fuel drop $W(r_S, t)$ is known.

The function $W(r_S, t)$ is obtained on the basis of the quasisteady diffusional combustion of a drop, under the assumption that the simplest reaction between fuel vapor and oxidant occurs in the flame. At high temper-

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atures in the combustion region, the drop temperature is little different from the boiling point T_S; large variations in T_{me} influencing the evaporation rate have little influence on the drop temperatures and T_S [2, 3]. Therefore, it will be assumed below that the drop temperature is constant at T_S. In [7, 8], an expression was obtained for the fuel flux, which, in the absence of convection and when the energy balance is taken into account for the external region $r > r_f$ ($r \to \infty$), may be written in the form

$$\frac{dM_T}{dt} = -r_S \left\{ \frac{4\pi\lambda_1}{c_1} \ln\left[1 + \frac{c_1}{L}(T_f - T_s)\right] + \frac{4\pi\lambda_2}{c_2 - c_{in}} \ln\left[1 + \frac{c_2 - c_{in}}{\beta}(T_f - T_{me})\right]\right\}, \tag{3}$$

where $\beta = Q_R - L - C_1(T_f - T_S)$.

In the absence of combustion ($T_f = T_{me}$), the situation corresponds to the case of drop evaporation in an inert medium [7, 8].

From Eq. (3), $W(r_S, t)$ may be obtained in the form of the product of two functions [6]

$$W(r_S, t) = \Omega(r_S)\,\omega(t), \tag{4}$$

where

$$\Omega(r_S) = \frac{1}{r_S}; \quad \omega(t) = -\left\{\frac{\lambda_1}{\rho_T c_1} \ln\left[1 + \frac{c_1}{L}(T_f - T_s)\right] + \frac{\lambda_2}{\rho_T (c_2 - c_{in})} \ln\left[1 + \frac{c_2 - c_{in}}{\beta}(T_f - T_{me})\right]\right\}.$$

The distribution function for the given model of combustion takes the form [6]

$$f(r_S, t) = \Sigma A_i r_S \exp\left[-a_i \frac{r_S^2}{2}\right] \exp\left[-a_i \int_0^t \omega(t)\,dt\right]. \tag{5}$$

When $t = 0$, the initial particle distribution over the radius is

$$f(r_S, 0) = f_0(r_S) = \Sigma A_i r_S \exp\left[-a_i \frac{r_S^2}{2}\right]. \tag{6}$$

To determine the self-similar parameters, following [6], the moment of order s is calculated

$$\langle r_S^s \rangle = \frac{N_0}{N(t)} \int_0^\infty r_S^s f(r_S, t)\,dr_S = \frac{N_0}{N(t)} \Sigma A_i 2^{s/2} a_i^{-\frac{s+2}{2}} \Gamma\left(\frac{s+2}{2}\right) \exp\left[-a_i \int_0^t \omega(t)\,dt\right]. \tag{7}$$

The proportion of unburnt fuel mass y(t) at time t may be defined as

$$y(t) = \frac{M_T(t)}{M_{T0}} = \frac{1}{\overline{r_{F0}^3}} \int_0^\infty r_S^s f(r_S, t)\,dr_S = \frac{\langle r_S^s \rangle N(t)}{\overline{r_{S0}^3} N_0}. \tag{8}$$

Then it follows from Eqs. (7) and (8) with s = 3 that

$$y(t) = \Sigma A_i 2^{3/2} a_i^{-5/2} \Gamma(5/2) (\overline{r_{S0}^3})^{-1} \exp\left[-a_i \int_0^t \omega(t)\,dt\right]. \tag{9}$$

Beginning at some time t_0 (small drops have vaporized, fuel vapor has entered into reaction with oxidant, ignition and partial combustion of the mixture in the kinetic region have occurred), there exists a distinctive kind of regular conditions of diffusional combustion, in which $f(r_S, t)$ and hence all the other functions will be determined only by the first term of the series

$$\langle r_S^s \rangle = \frac{N_0}{N(t)} A 2^{s/2} a^{-\frac{s+2}{2}} \Gamma\left(\frac{s+2}{2}\right) \exp\left[-a \int_{t_0}^t \omega(t)\,dt\right], \tag{10}$$

and from the normalization condition (s = 0) it is found that

$$N(t) = \frac{A}{a} N_0 \exp\left[-a \int_{t_0}^t \omega(t)\,dt\right]. \tag{11}$$

The self-similar parameters are found from Eqs. (8), (10), and (11)

$$\langle r_{\mathrm{S}}^{s} \rangle = \overline{r_{\mathrm{S}.0}^{s}} = 2^{s/2} a^{-s/2} \Gamma\left(\frac{s+2}{2}\right). \tag{12}$$

From Eq. (12) with s = 1, the following expression is obtained for a

$$a = \frac{2\Gamma^{2}(3/2)}{(\overline{r_{\mathrm{S}.0}})^{2}}; \quad \langle r_{\mathrm{S}}^{s} \rangle = \overline{r_{\mathrm{S}0}^{s}} = \frac{(\overline{r_{\mathrm{S}0}})^{s} \Gamma\left(\frac{s+2}{2}\right)}{\Gamma^{s}(3/2)}. \tag{13}$$

It follows from Eqs. (6) and (2) that $a \approx \mathrm{A}$. Then, the proportion of unburnt fuel mass is determined from Eq. (9), taking account of Eqs. (8), (11), and (13)

$$y(t) = \exp\left[-a \int_{0}^{t} \omega(t)\, dt\right]. \tag{14}$$

Dependences for y(t), $T_{\mathrm{me}}(t)$, and so on may be obtained from a consideration of the thermal and material balances in the combustion of a liquid-fuel drop taking account of certain assumptions.

In Eq. (4) for $\omega(t)$, the flame temperature T_{f} in the general case depends on T_{me}, the oxidant concentration in the gas, its diffusion coefficient and other parameters, and therefore the given problem is not solved in final form, and requires the use of numerical calculation. However, it follows from diffusional theory that the temperature difference between the flame and the medium $(T_{\mathrm{f}} - T_{\mathrm{me}})$ for combustion in air diluted by inert gas is up to 90% of the maximum possible $(T_{a} - T_{\mathrm{me}})$ [2]. This means that the temperature T_{f} differs only slightly from the theoretical T_{a} and may be defined in terms of it [2, 3]. The temperature T_{f} increases with increase in temperature of the medium, and decreases as the oxygen is consumed [2, 7], and therefore, it will change only slightly and may be regarded as known and constant; this is acceptable for engineering calculations of the combustion process in the diffusional region.

The thermal balance of the system is written in the form

$$[(M_{\mathrm{in}} c_{\mathrm{in}} + M_{\mathrm{ox}} c_{\mathrm{ox}}) + M c_{2}] \frac{d\vartheta}{dt} = \frac{dM_{\mathrm{T}}}{dt} [\beta + (c_{2} - c_{\mathrm{in}})(\vartheta_{0} - \vartheta)], \tag{15}$$

where $\vartheta_{0} - \vartheta = (T_{\mathrm{f}} - T_{\mathrm{me}.0}) - (T_{\mathrm{me}} - T_{\mathrm{me}.0}) = T_{\mathrm{f}} - T_{\mathrm{me}}$.

Introducing the notation

$$\mu = \frac{M_{\mathrm{T}0}}{M_{\mathrm{in}}}; \quad c_{\mathrm{pr}} = \frac{M_{\mathrm{ox}}}{M} c_{\mathrm{ox}} + c_{2},$$

Eq. (15) may be reduced to the form

$$\left[1 + \mu \frac{c_{\mathrm{pr}}}{c_{\mathrm{in}}} (1 - y)\right] d\vartheta = \frac{\mu}{c_{\mathrm{in}}} [\beta + (c_{2} - c_{\mathrm{in}})(\vartheta_{0} - \vartheta)]\, dy. \tag{16}$$

Solution of Eq. (16) with the initial conditions t = 0, y = 1, $\vartheta|_{t=0} = \vartheta_{0}$ gives

$$y = 1 - \frac{c_{\mathrm{in}}}{\mu c_{\mathrm{pr}}} \left\{\left[1 + \frac{c_{2} - c_{\mathrm{in}}}{\beta} (\vartheta_{0} - \vartheta)\right]^{\frac{c_{\mathrm{pr}}}{c_{2} - c_{\mathrm{in}}}} - 1\right\}. \tag{17}$$

An equation for y(t) is obtained from Eq. (14), taking account of Eq. (17)

$$\frac{dy}{dt} + ky + b \frac{c_{2} - c_{\mathrm{in}}}{c_{\mathrm{pr}}} \ln\left[1 + \mu \frac{c_{\mathrm{pr}}}{c_{\mathrm{in}}} (1 - y)\right] y = 0, \tag{18}$$

where

$$k = \frac{a\lambda_{1}}{\rho_{\mathrm{T}} c_{1}} \ln\left[1 + \frac{c_{1}}{L} (T_{f} - T_{s})\right]; \quad b = \frac{a\lambda_{2}}{\rho_{\mathrm{T}} (c_{2} - c_{\mathrm{in}})}.$$

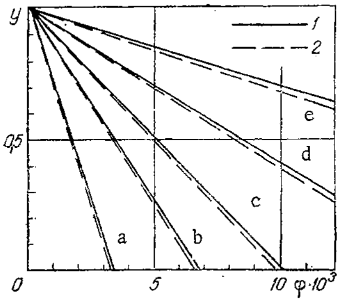

Fig. 1

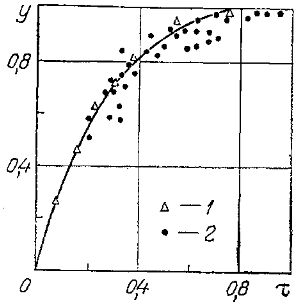

Fig. 2

Fig. 1. Graphs of $y(\varphi)$: 1) calculation from linearized Eq. (19); 2) from accurate Eq. (17); a) $\mu = 0.01$; b) 0.02; c) 0.03; d) 0.05; e) 0.1; $\varphi = [(c_2 - c_{in})/\beta](\vartheta_0 - \vartheta)$.

Fig. 2. Combustion kinetics of a polydisperse system of fuel drops: the curve corresponds to numerical calculations [9]; 1) calculations from Eq. (25); 2) experimental data [9, 10] for various forms of fuel; $\tau = t/t_{max}$; t_{max} is determined at $y = 0.01$.

Numerical methods must be used to solve Eq. (18); however, it may be reduced to quadratures if the natural logarithm is expanded in Taylor series, retaining only a linear approximation in μ. This is equivalent to linearization of the second logarithm in the expression for the time component of the combustion rate $\omega(t)$ in Eq. (4), which is an analog of transition to calculation of the combustion by the "reduced-film" method [2] and allows the relations obtained to be used for combustion processes with convective heat transfer. A linearized equation for $y(\vartheta)$ may be obtained from Eq. (17), but only with loss of some of the information on the influence of the specific heat of the combustion products on the relation between y and ϑ; therefore, it is more expedient to obtain $y(\vartheta)$ from the heat balance of the drop under the assumption that the specific heat of the gas mixture does not depend on y and the heat consumed in heating the gas in the external layer bounding the flame front is negligibly small.

Then

$$y = 1 - \frac{\vartheta_0 - \vartheta}{\vartheta^*}, \tag{19}$$

where $\vartheta^* = \mu_g \beta / c_g$; $\mu_g = M_{T0}/M_g$, and the equation for y takes the form

$$\frac{dy}{dt} + (k + m)\, y - m y^2 = 0, \tag{20}$$

where

$$m = b\, \frac{c_2 - c_{in}}{c_{in}}\, \mu, \quad \mu = \frac{a\lambda_2}{\rho_T c_g}\, \mu_g.$$

Calculations from Eqs. (17) and (19) show (Fig. 1) that, over the whole range of fuel concentration, taking account of complete burnup (for $\mu_g \leq 0.1$), Eq. (20) may be used; the maximum error is no more than 5%.

With the initial condition $y(0) = 1$, the solution of Eq. (20) takes the form

$$y = (k + m)\,[k \exp(k + m)\, t + m]^{-1}. \tag{21}$$

It follows from Eqs. (19) and (21), in a linear approximation, that

$$T_{me} = T_{me0} + \vartheta^* \{1 - (k + m)\,[k \exp(k + m)\, t + m]^{-1}\}. \tag{22}$$

The temperature of the medium may be more accurately obtained from Eqs. (17) and (21) but it is not given here because of its complexity.

All the basic conclusions of combustion kinetics are also valid for a steady gas flux including burning droplets and moving at velocity V(x). Writing the kinetic equation for the particle distribution function over the radius [6], solving it for self-similar combustion conditions, and drawing all the subsequent conclusions, a differential equation is obtained for y(x)

$$\frac{dy}{dx}+\frac{k+m}{V(x)}\,y-\frac{m}{V(x)}\,y^2=0, \tag{23}$$

with a solution in the form

$$y=(k+m)\left[k\exp\int\limits_0^x\frac{k+m}{V(x)}\,dx+m\right]^{-1}. \tag{24}$$

Thus, Eqs. (17)-(22) may be used to calculate combustion processes in a steady gas flux or for the case of a moving cloud of droplets if, as is evident from a comparison of Eqs. (21) and (24), the following substitution is made [6]

$$t=\int\limits_0^x\frac{dx}{V(x)}.$$

The method proposed for calculating the combustion kinetics of a polydisperse system of liquid-fuel drops is in good agreement with experimental data [9, 10] obtained for the ignition of atomized fuel in a recording chamber (Fig. 2). Calculation for $\bar{r}_{F0}$ = 100 μm; λ_1 = 0.06 J/msec · K; λ_2 = 0.08 J/msec · K; L = 189 kJ/kg; c_1 = 2 kJ/kg · K; c_2 = 3 kJ/kg · K; T_f = 1973°K; T_S = 426°K; μ = 0.12 kg/kg; ρ_T = 824 kg/m^3 for aircraft diesel fuel yields good agreement with the theoretical data of [9] obtained by numerical integration (Fig. 2).

For the above parameters, y(t) is given by the simple expression

$$y=16.58(15.97\exp 16.58t+0.61)^{-1}, \tag{25}$$

which, in turn, may be simplified.

Analogous calculations for calculating the combustion kinetics of a polydisperse fuel in a turbulized gas may be obtained on the basis of "reduced-film" theory, assuming that the heat-transfer coefficient from the gas to the drop surface is known in advance, i.e., it may be calculated, in the first approximation, from the well-known formula of [2].

All the conclusions and equations obtained in this case will completely coincide with those in [6].

NOTATION

r_f, r_S, radius of flame front and drop; $f(r_S, t)$, drop distribution function over the radius; $W(r_S, t)$, combustion rate; N, N_0, current and total numbers of drops initially; A_i, a_i, constants; λ_1, λ_2, thermal conductivity of the gas in the regions in front of and behind the flame front; c_1, c_2, c_{in}, c_{pr}, c_{ox}, c_g, isobaric specific heat of the fuel vapor in regions in front of and behind the flame front, of inert gas, the combustion products, oxygen, and the gas mixture; T_S, T_f, T_{me}, $T_{me.0}$, temperature of the drop surface, the flame front, and the mean current and initial medium outside the combustion region; L, latent heat of vaporization of the fuel; Q_R, heat of combustion; ρ_T, density of the liquid fuel; Γ, gamma function; M_T, M_{T0}, M_{ox}, M_{in}, M, M_g, mass of the current and initial drop, oxygen, inert gas, fuel vapor, and gas mixture; μ, μ_g, initial fuel concentration in inert gas and in the total mixture (kg fuel/kg dry gas). Indices: S, drop surface; f, flame front; in, inert gas; pr, combustion products; g, gas mixture; me, medium; 0, initially.

LITERATURE CITED

1. T. V. Vilenskii and D. M. Khzmalyan, Dynamics of Atomized-Fuel Combustion [in Russian], Énergiya, Moscow (1978).
2. G. F. Knorre et al., Theory of Furnace Processes [in Russian], Énergiya, Moscow–Leningrad (1966).
3. V. V. Pomerantsev et al., Principles of the Practical Theory of Combustion [in Russian], Énergiya, Leningrad (1973).
4. M. V. Buikov, "Evaporation of polydisperse mist," Kolloidn. Zh., 24, No. 4, 390-395 (1962).
5. Yu. A. Buevich, "Mass-transfer kinetics of a polydisperse particle system with the surrounding medium," Prikl. Mat. Tekh. Fiz., No. 1, 50-57 (1966).
6. G. P. Yasnikov, "Kinetics of self-similar conditions of the evaporation of a polydisperse system of drops," Inzh.-Fiz. Zh., 42, No. 2, 243-250 (1982).

7. B. I. Khaikin, "Heterogeneous combustion," in: Heat and Mass Transfer in Combustion Processes [in Russian], OIKhF, Chernogolovka (1980), pp. 58-79.
8. S. Kumagai, Combustion [in Russian], Khimiya, Moscow (1979).
9. R. S. Tyul'panov and O. P. Sobolev, "Combustion of a polydisperse liquid-fuel torch," Fiz. Goreniya Vzryva, No. 1, 94-97 (1967).
10. R. S. Tyul'panov, "Burnup of atomized hydrocarbon fuel in blast furnaces," Fiz. Goreniya Vzryva, No. 1, 88-99 (1966).

DISTRIBUTION OF THE HEAT FLUX TO THE CALORIMETRIC PROBE IN A MULTIARC PLASMA REACTOR

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UDC 533.9.002.5:661.632-492:536.244

The article presents the distribution of the heat flux to the calorimetric probe in a multiarc plasma reactor.

The quality of treatment of materials in multiarc plasma reactors is decisively influenced by the speed with which the plasma jets mix [1, 2]. It is known that the heat flux from the gas to the calorimetric probe is a function of the temperature and of the speed of the gas. Therefore the nature of its distribution gives some indication of the speed with which the plasma jets mix.

The present work represents an investigation of the distribution of the heat flux from the gas to the calorimetric probe in a widely used plasma reactor [3, 4] in dependence on the flow rate of the plasma-forming gas and the power supplied to the plasmatrons.

The experimental device (Fig. 1) consists of the plasma reactor 1 with three plasmatrons 2 and linear calorimetric probe (LCP) 3 mounted in the upper part of the plasma reactor. In the lateral wall of the reactor along its generatrix at an angle of 0, 30, and 60° there are three rows of holes with dielectric inserts 4. The plasmatrons are of the electric-arc type, with linear arrangement and self-adjusting arc length, and their channel diameter is $8 \cdot 10^{-3}$ m. The overall electric power used for the electric discharge varied between 40 and 60 kW. The overall flow rate of plasma-forming gas—air was varied within the limits $1.5 \cdot 10^{-3}$ to $4.5 \cdot 10^{-3}$ kg/sec. Raw material was not charged into the reactor. The arrangement of the LCP and the method of measuring the heat flux were described in detail in [5, 6].

Figure 2 shows the distribution of the heat flux in the diametral section of the reactor $3 \cdot 10^{-2}$ and $7 \cdot 10^{-2}$ m from the point of intersection of the longitudinal axes of the plasmatrons. The LCP entered the reactor at the angle $\alpha = 60°$ (Fig. 1). Analogous distributions of the heat flux were obtained with α equal to 0 and 30°.

An analysis of the obtained data showed that in the cross section of the reactor at a distance of 0.3 of the bore from the imaginary point of intersection of the axes of the plasmatrons (point A) the distribution of the heat fluxes is very nonuniform. With decreasing power supplied as well as with decreasing flow rate of plasma-forming gas the gradient of the heat flux over the radius decreases. However, this also entails a decrease of the fraction of the longitudinal section of the reactor occupied by flux with high enthalpy. With increasing distance from the place of collision of the plasma jets the heat fluxes are further equalized in the cross sections of the reactor.

It follows from the oscillograms that at a distance of $3 \cdot 10^{-2}$ m from point A there are one-dimensional flows directed toward the periphery of the reactor. It may be assumed that these flows originate in conse-

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Dnepropetrovsk Institute of Chemical Technology. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 45, No. 3, pp. 458-461, September, 1983. Original article submitted May 4, 1982.

0022-0841/83/4503-1042$07.50